



TITLE:

Subsequences of Normal Sequences (エルゴード理論とその周辺)

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Subsequences of normal sequences

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Let Σ be any compact metric space. Let $N=\{0,1,2,\dots\}$ be the set of non-negative integers. By Σ^N , we mean the product space of Σ with the product topology. The i -th coordinate ($i \in N$) of $\alpha \in \Sigma^N$ is denoted by $\alpha(i)$. An element of Σ^N is called a sequence. Let T be the shift on Σ^N ; $(T\alpha)(i)=\alpha(i+1)$ for any $\alpha \in \Sigma^N$ and $i \in N$.

By a measure on a topological space, we always mean a probability Borel measure. Let W be an arbitrary compact space and let ν_n ($n=0,1,\dots$) and ν be measures on W . We say that ν_n converges weakly [11] to ν as $n \rightarrow \infty$, and denote $w\text{-}\lim_{n \rightarrow \infty} \nu_n = \nu$, if for any real-valued continuous function f on W , $\int f d\nu_n$ converges to $\int f d\nu$ as $n \rightarrow \infty$. For $x \in W$, δ_x is the unit measure at x . By a non-degenerate measure on W , we mean a measure which is not a unit measure.

Let $\alpha \in \Sigma^N$. Let E_α denote the family of all infinite subsets S of N such that

$$(1.1) \quad \mu_\alpha^S = w\text{-}\lim_{\substack{n \in S \\ n \rightarrow \infty}} \frac{1}{n} \sum_{i=0}^{n-1} \delta_{T^i \alpha}$$

exists. Note that $E_\alpha \neq \emptyset$ for any $\alpha \in \Sigma^N$ since the space of measures on a compact metric space is compact in the topology of weak convergence (see [11]). Also, note that μ_α^S is a T -invariant measure for any $\alpha \in \Sigma^N$ and $S \in E_\alpha$. We call $\alpha \in \Sigma^N$ a stochastic sequence [3] (or sometimes a quasi-regular point in Σ^N with respect to T [9]) if $N \in E_\alpha$. In this

case, we denote $\mu_\alpha = \mu_\alpha^N$. Let P be a non-degenerate measure on Σ . A stochastic sequence $\alpha \in \Sigma^N$ is called a P-normal sequence if μ_α equals P^N , the product measure of P on Σ^N . Note that if $\Sigma = \{0, 1, \dots, r-1\}$ and $P(\{i\}) = 1/r$ for $i = 0, 1, \dots, r-1$, then the notion of P-normal sequence coincides with the usual notion of r-adic normal sequence. The set of all P-normal sequence is denoted by Nor_P . A strictly increasing function from N to N is called a selection function. Let τ be a selection function. For $\alpha \in \Sigma^N$, the subsequence of α selected by τ is defined by $(\alpha \circ \tau)(i) = \alpha(\tau(i))$ for any $i \in N$ and denoted by $\alpha \circ \tau$. Following von Mises, $\alpha \in \Sigma^N$ is called a τ -collective if

$$(1.2) \quad \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{i=0}^{n-1} \delta_{\alpha(i)} = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{i=0}^{n-1} \delta_{\alpha(\tau(i))}.$$

Our problem is to characterize a selection function τ which satisfies the following conditions. Denote $\text{Nor}_{P \circ \tau} = \{\alpha \circ \tau; \alpha \in \text{Nor}_P\}$.

Condition 1. Any $\alpha \in \text{Nor}_P$ is a τ -collective.

Condition 2. $\text{Nor}_{P \circ \tau} \subset \text{Nor}_P$

Condition 3. $\text{Nor}_{P \circ \tau} = \text{Nor}_P$

Clearly, condition 3 implies condition 2. It is also easy to verify that condition 2 implies condition 1.

In this paper, we prove that each of the above three conditions is equivalent to condition 4 stated below under a reasonable restriction that $\overline{\lim}_{n \rightarrow \infty} \frac{\tau(n)}{n} < \infty$ (Theorem 4). It should be remarked that the fact that condition 4 implies condition 2 under the restriction stated above was already obtained by Benjamin Weiss [14] in 1971.

To state condition 4, some more notions are necessary. For a selection function τ , denote by $\theta_\tau \in \{0,1\}^N$ the 0-1-sequence defined by

$$(1.3) \quad \theta_\tau(i) = \begin{cases} 1 & \text{if } i \in \{\tau(j); j \in N\} \\ 0 & \text{else} \end{cases} \quad (i \in N).$$

That is, $\theta_\tau(i) = 1$ if and only if the i -th coordinate is selected as a subsequence by the selection function τ . For a T -invariant measure μ on $\{0,1\}^N$, where T is the shift on $\{0,1\}^N$, the entropy of the measure-preserving transformation T on the measure space $(\{0,1\}^N, \mu)$ is denoted by $h_\mu(T)$. That is,

$$(1.4) \quad h_\mu(T) = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{\xi \in \{0,1\}^n} -\mu(\Gamma_\xi) \cdot \log \mu(\Gamma_\xi),$$

where for $\xi = (\xi_0, \xi_1, \dots, \xi_{n-1}) \in \{0,1\}^n$,

$$(1.5) \quad \Gamma_\xi = \{\beta \in \{0,1\}^N; \beta(i) = \xi_i \text{ for } i=0,1,\dots,n-1\}.$$

The above limit is known to exist [2]. Following [14], $\beta \in \{0,1\}^N$ is said to be completely deterministic if $h_\mu(T)=0$ for any $\mu \in \{\mu_\beta^S; S \in \Sigma_\beta\}$. Now, we state condition 4.

Condition 4. θ_τ is completely deterministic.

Note that condition 4 is indifferent to what Σ and P are. This condition is not only simple but also easy to check. Various types of sequences are known to be completely deterministic (Example 1).

ON KOLMOGOROV'S COMPLEXITY AND INFORMATION

TETURO KAMAE

About Kolmogorov's complexity measure K , we prove the following theorem, which seems rather eccentric.

Theorem. *For any C , there exists a sentence y such that*

$$K(y) - K(y|x) > C$$

holds except for finitely many sentences x .